A SIMPLE CRITERION FOR VORTEX BREAKDOWN

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ABSTRACT

A mathematical model of vortex breakdown is developed for a free cylindrical vortex of radius "a" that passes through a region of non-cylindrical flow and regains the cylindrical flow structure of radius "b" downstream. The criterion of zero axial velocity within the free vortex, downstream, yields a relationship between the radius ratio b/a and the reduced frequency of the upstream vortex for vortex breakdown. The limiting value of the upstream reduced frequency correlates well with recent published experimental data.

NOMENCLATURE

a	upstream vortex radius
b	downstream vortex radius
c	root chord of delta wing
f	frequency
J_0	Bessel function of first kind, order 1
J_1	Bessel function of first kind, order 0
k	$=2\Omega / U$
s	semispan of delta wing
s^*	local semispan
U	free stream axial velocity
U_s	slipstream axial velocity
u	axial velocity
v	radial velocity
w	circumferencial velocity
x	chordwise location, delta wing
α	helix angle of velocities
β	helix angle of vorticities
ω_x	axial vorticity
ω_ϕ	azimuthal vorticity
Ω	angular velocity of upstream vortex.
σ	field radius

INTRODUCTION

Brown & Lopez (1990), Darmofal (1992) among others, have studied the phenomenon of axisymmetric vortex breakdown. A necessary condition for vortex breakdown postulated by Brown & Lopez is the generation of zero axial velocity. They demonstrate that an axisymmetric vortex which increases in size generates negative azimuthal vorticity, which in turn decelerates the axial flow within the core of the vortex. Jumper et.al.,(1993) utilized these concepts to postulate a simple theory for vortex breakdown. Darmofal (1992) examined the origins of negative azimuthal vorticity and reasoned vortex tilting and stretching as the primary mechanisms responsible for generating the negative azimuthal vorticity.

ANALYSIS

In the present study, we investigate the relation between the vortex strength and the vortex size at breakdown. We consider a free vortex embedded in an irrotational flow, which over some portion of it's length upstream is cylindrical. The fluid within the vortex passes through a region of non-cylindrical flow and regains the cylindrical structure, at some location, downstream. In the upstream region, the cylindrical vortex of radius "a" is assumed to have a uniform axial velocity U, see Figure 1. The circumferencial velocity upstream is assumed to be that of a classical free vortex, viz., $w \propto \sigma$ for $\sigma \leq a$ and $w \propto 1/\sigma$ for $\sigma \geq a$. In the down stream region, the vortex has a

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different radius "b" . The vortex here is assumed to have uniform axial velocity U_s and the circumferencial velocity component in the down stream region is assumed to vary inversely with the radius, in the irrotational outer region.

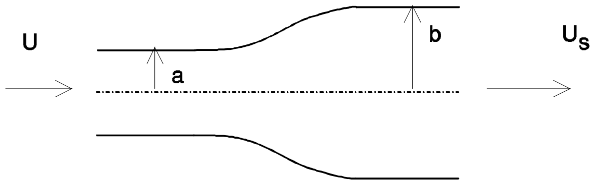

Figure 1:Schematic of a Vortex Increasing in Diameter

Although the model represented above may appear too restrictive to be of any practical interest, experimental observations on vortices emerging from thin slender delta wings and leading edge extensions of high performance airplanes at high angles of attack show a vortex pattern that is similar to the one considered here. The leading edge vortex once formed from the leading edge feeding sheet can be considered to be essentially a free vortex, with minimal interaction with the wing. Figure 2 shows the vortex emanating from the leading edge extension of a F-18 airplane model in the water tunnel at an angle of 34°.

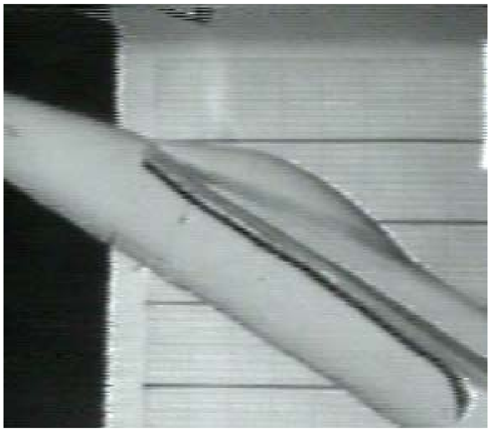

Figure 2:Leading edge extension vortex on a 1/48 scale model of F-18 in Water Tunnel. (Reynold's number ~25000, α =34°)

Following the analysis of Batchelor (1967), the axial velocity distribution within the vortex downstream is given by:

$$\frac{u}{U} = 1 + \left(\frac{a^2}{b^2} - 1\right)\frac{kb\,J_0(k\sigma)}{2J_1(kb)} \qquad (1)$$

and the helix component of the velocity is

$$\frac{w}{\Omega\sigma} = 1 + \left(\frac{a^2}{b^2} - 1\right)\frac{b\,J_1(k\sigma)}{\sigma\,J_1(kb)} \qquad (2)$$

Here, $k = \frac{2\Omega}{U}$ and Ω is the angular velocity of the vortex. J_0 and J_1 represent the Bessel's function of the first kind, order 0 and 1 respectively. σ represents the radius where the axial or azimuthal velocity is being reckoned. The axial component of the vorticity ω_X, is given by:

$$\omega_X = \frac{1}{\sigma}\frac{\partial}{\partial\sigma}(\sigma\, w) \; . \qquad (3)$$

Substituting Equation (2) in (3) yields,

$$\omega_x = 2\Omega\left[\, 1 + \left(\frac{a^2}{b^2} - 1\right)\frac{\Omega b}{U}\,\frac{J_0(k\sigma)}{J_1(kb)}\,\right] \qquad (4)$$

The azimuthal component of vorticity is given by:

$$\omega_\phi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial\sigma} \qquad (5)$$

In the fully developed downstream region of the vortex, the first term in Equation (5) will be zero. Thence,

$$\omega_\phi = 2\Omega\left[\frac{a^2}{b^2} - 1\right]\left[\frac{\Omega b}{U}\right]\frac{J_1(k\sigma)}{J_1(kb)} \; . \qquad (6)$$

We observe from equations (4) and (6) that as the downstream vortex radius increases, the axial vorticity decreases and the negative azimuthal vorticity increases.

The helix angle α for velocities ($\tan\alpha = w / u$) and the helix angle for vorticities ($\tan\beta = \frac{\omega_\phi}{\omega_x}$) may now be written from equations (1),(2),(4) and (5) as:

$$\tan\alpha = \frac{\Omega\, b}{U}\;\frac{1 + \left(\frac{a^2}{b^2} - 1\right)\frac{b}{\sigma}\,\frac{J_1(k\sigma)}{J_1(kb)}}{1 + \left(\frac{a^2}{b^2} - 1\right)\frac{kb}{2}\,\frac{J_0(k\sigma)}{J_1(kb)}} \qquad (7)$$

and

$$\tan\beta = \frac{\frac{\Omega\,b}{U}\left(\frac{a^2}{b^2} - 1\right)\frac{J_1(k\sigma)}{J_1(k\mathrm{b})}}{1 + \left(\frac{a^2}{b^2} - 1\right)\frac{kb}{2}\frac{J_0(k\sigma)}{J_1(k\mathrm{b})}} \quad . \qquad (8)$$

We now postulate, following Jumper et. al., (1993), that vortex breakdown occurs when the axial velocity in the vortex down stream approaches zero. In terms of the helix angle for velocities, α, this is equivalent to the helix angle approaching $\pi / 2$ for vortex breakdown. We observe from Equation (8) that this condition also implies that the helix angle for vorticies, β, approaches $\pi / 2$ simultaneously. Thus, from Equation (7) or (8), we write the relation between the radius ratio a / b and the vortex strength for vortex breakdown as

$$\frac{a}{b} = \sqrt{1 - \frac{2J_1(k\mathrm{b})}{k\mathrm{b}}} \qquad (9)$$

Figure 3 depicts a plot of Equation (9). We observe from Figure 3 that vortices of higher strength "breakdown" at relatively low values of b / a , whereas, vortices of lower strength breakdown at relatively high values of b / a. We also observe that at a value of radius ratio of b / a = 1, the value of *k*a is 3.832. This implies that the upstream reduced frequency of the free vortex, Ωa/U, cannot exceed 1.916, for the free vortex to experience a region of non-cylindrical flow of increasing diameter. It is unlikely that this value of ka will be realized in practice. Reported experimental data by Gursul (1994) and Visser and Nelson (1993) confirm this statement. While these experiments refer to leading edge vortices over delta wings, it is interesting to estimate the upper bound for *k*a from experimental data. In Gursul's experiments, the maximum dimensionless dominant frequency, f c/U on a 60° delta wing at x/c =0.89 was 2.(see Figure 7 of Gursul, 1994). Assuming that the radius of the upstream vortex a ~ 0.1 x, we obtain the maximum value of Ωa/U in Gursul's experiments to be 1.3. In Visser and Nelson's data (see figure 5 of Visser & Nelson, 1993)the maximum dimensionless axial vorticity $\Omega s^*/U$ on a 75° delta wing at an angle of attack of 20° at x/c =0.5, is found to be 70. Once again assuming that the radius of the core of the upstream vortex a ~ 0.1x, we obtain the maximum value of Ωa/U in Visser and Nelson's experiments to be 1.6. These upper bounds of Ωa/U from experimental data are given for preliminary validation of the results of the present mathematical analysis. It must however be emphasized here that the analytical model assumes inviscid cylindrical flow, whereas, the experimental results are for a viscid vortex over delta wings. Nonetheless, the present comparision of the upper limit for the reduced frequency of the upstream vortex between the present mathematical analysis and experiments should warrant further investigation.

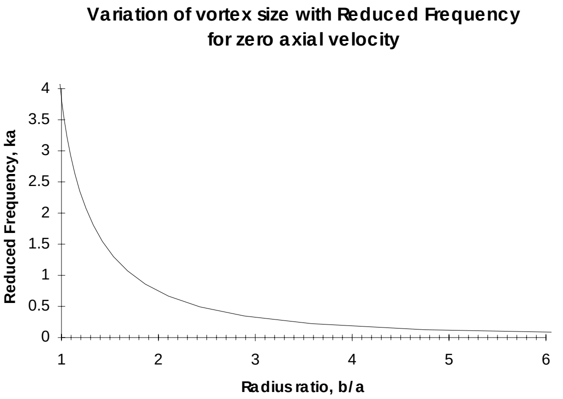

Figure 3: Variation of radius ratio b/a with reduced frequency *k*a for zero axial velocity

The relation given by Equation (9) may be used for predicting vortex breakdown, as follows. The reduced frequency (Ω a / U = *k*a / 2) of the upstream flow is determined by the angle of attack and sweep for a thin slender delta wing. Thus, if the upstream reduced frequency is known, one can "predict" the location of the vortex breakdown, by measuring the downstream vortex size. Conversely, if b / a at breakdown is known, one can "estimate" the upstream vortex frequency, Ω.

From Equation (1) we can plot the axial velocity variation in the downstream region along the radius. Figure 4 shows the variation of axial velocity as a function of $k\sigma$ for kb =2, b/a =1.537. We observe from this figure that the axial velocity is finite and positive away from the center of the vortex, eventhough the axial velocity at the axis itself is zero.

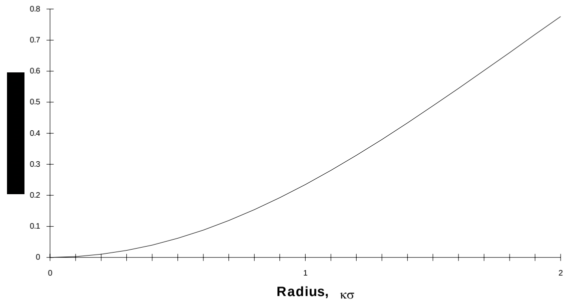

Figure 4: Axial Velocity Variation along the Radius of the downstream vortex. kb=2, b/a=1.537.

From Equation (7) and (8) , for σ = b, the relation between the helix angle for velocities and helix angle for vortices may be written as:

$$\frac{\tan\,\alpha}{\tan\beta} = \frac{1}{\frac{a^2}{b^2} - 1} + 1. \qquad (10)$$

We observe from Equation (10) that for b > a, the helix angle for velocities has the opposite sign of the helix angle for vortices. Following the discussions put forth by Brown & Lopez (1990), we observe from the present analysis that the azimuthal vorticity in the downstream region will be negative for positive values of helix angle for velocities and axial vorticity in the downstream region.

CONCLUSIONS

Through a simple mathematical model of a cylindrical vortex passing through a region of non-cylindrical flow which regains the cylindrical flow structure down stream, a relation between the reduced frequency of the upstream vortex and the ratio of radii b/a of the vortices, is established for vortex breakdown.The upper bound for the reduced frequency of the upstream vortex predicted by the present model correlates well with existing experimental results in the literature. It is also observed that the vortex structure in the downstream region is preserved, eventhough the axial velocity along the vortex axis itself is zero. The present analysis clearly supports the hypothesis put forth by Brown & Lopez (1990) viz., the existence of negative azimuthal vorticity in the flow for vortex breakdown.

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